ON A CYLINDER THE INTERSECTION OF WHICH WITH A SPHERE WILL DEVELOP INTO AN ELLIPSE.*

By D. N. LEHMER, University of California.

1. In his *Mathematical Recreations* published about 1674 Van Etten describes a method of drawing an "oval" with a pair of compasses leaving the distance between the points of the instrument unaltered. His scheme was to stretch the drawing paper around a cylinder. The curve thus obtained is not an ellipse but a transcendental curve whose equation is

$$y^2+4b^2\sin^2\frac{x}{2b}=a^2,$$

where b is the radius of the cylinder and a the distance between the points of the compasses. The curve is clearly the developed intersection of a sphere of radius a with a cylinder of radius b, the center of the sphere being on the surface of the cylinder.

2. The question suggests itself to find the cylinder such that the curve as obtained above will develop into an ellipse. Take the origin at the center of the sphere and let the y-axis be an element of the cylinder. Let a plane parallel to the xz-plane cut out on the cylinder a curve of length s. Then the equation of the ellipse in question will be

$$\frac{y^2}{a^2}+\frac{s^2}{b^2}=1.$$

To find the nature of the cylinder we have then the equations

$$x^2+y^2+z^2=a^2 \text{.........} (1),$$

$$\frac{y^2}{a^2}+\frac{s^2}{b^2}=1 \text{.........} (2),$$

$$\frac{ds}{ax}=\sqrt{1+\frac{dz^2}{ax^2}} \text{..........} (3).$$

From these it is easy to derive the differential equation connecting x and z:

$$a^2(x^2+z^2)\left(1+\frac{dz^2}{ax^2}\right)=b^2\left(z\frac{dz}{ax}+x\right)^2.$$

Changing to polar coördinates and reducing,

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$$\frac{d\rho}{d\theta}=\pm\frac{a\rho}{\sqrt{(b^2-a^2)}}, \text{ or } \kappa\rho=e^{\pm[a/\sqrt{(b^2-a^2)}]\theta}.$$

The cylinder has therefore for its right section a logarithmic spiral or rather two arranged in opposite directions.

3. This curious result may be stated in the following way. If an elliptical hole be cut out of a sheet of paper and the sheet wrapped about a sphere the diameter of which equals the minor axis of the ellipse the sheet wraps up into a cylinder a cross section of which is a logarithmic spiral. A model illustrating this relation between the ellipse and the spiral is easily constructed.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

200. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

No matter what value x be given, the *numerical* value of the expression $(x+2)/(2x^2+3x+6)$ can never exceed $\frac{1}{3}$.

II. Solution by G. W. GREENWOOD, M.A., Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

We know that $y+1/y \nless 2$.

$\therefore z/2+2/z \nless 2$. $2z+8/z \nless 8$. $2z+8/z-5 \nless 3$. $\dfrac{2z^2-5z+8}{z} \nless 3$.

Now put $z=x+2$ and we get

$$\frac{2x^2+3x+6}{x+2} \nless 3;\ i.\ e.,\ \frac{x+2}{2x^2+3x+6} \ngtr \tfrac{1}{3}.$$

202. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Express in the form of radicals the roots of the equation

$$x^9+9mx^7+27m^2x^5+30m^3x^3+9mx+2r=0.$$

I. Solution by A. H. HOLMES, Brunswick, Maine.

Writing $x^3+3mx=y$ the equation becomes

$$y^3+3m^3y+2r=0\text{..........}(1).$$